Gravitační pole

238/ A1

F_g / N	m / kg	a_g / m s^{-2}
250	25	? (10)
(120) ?	5	24
128	? (80)	1,6

$$F_g = \kappa \frac{m_1 m_2}{r^2}$$

$$a_g = \frac{\kappa m_2}{r^2} = \frac{\kappa M_z}{R_z^2}$$

a) $a_g = \frac{F_g}{m} = \frac{250}{25}\ \text{m s}^{-2} = 10\ \text{m s}^{-2}$

b) $F_g = m \cdot a_g = 5 \cdot 24\ \text{N} = 120\ \text{N}$

c) $m = \frac{F_g}{a_g} = \frac{128}{1,6}\ \text{kg} = 80\ \text{kg}$

A2/

$a_{g_0} \doteq 10\ \text{m s}^{-2}$ pro R_z

$a_g = ?\ (\text{m s}^{-2})$ pro a) $r = 2R_z$

b) $r = \sqrt{2}\, R_z$

$$a_g \sim \frac{1}{R_z^2}$$

a) $a_g = \frac{\kappa M_z}{(2R_z)^2} = \frac{\kappa M_z}{4R_z^2} = \frac{1}{4} a_{g_0}$

$a_g = \frac{1}{4} \cdot 10\ \text{m s}^{-2} = \underline{2,5\ \text{m s}^{-2}}$

b) $a_g = \frac{\kappa M_z}{(\sqrt{2} R_z)^2} = \frac{\kappa M_z}{2R_z^2} = \frac{1}{2} a_{g_0}$

$a_g = \frac{1}{2} \cdot 10\ \text{m s}^{-2} = \underline{5\ \text{m s}^{-2}}$

239/4

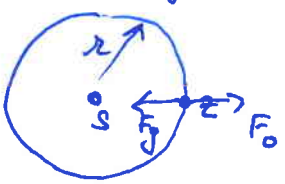
$r = 150 \cdot 10^6\ \text{km} = 150 \cdot 10^9\ \text{m}$

$T = 365\ \text{dní} = 31\,536\,000\ \text{s}$

a) $a = ?\ (\text{m s}^{-2})$

b) $M_\odot = ?\ (\text{kg})$

- dráha planety je přibližně kruhová $\Rightarrow$



$$F_g \doteq F_o$$

$$\kappa \frac{M_z M_\odot}{r^2} = \frac{M_z \cdot v^2}{r} \Rightarrow M_\odot = \frac{v^2 r}{\kappa}$$

b)

- rychlost Země určíme

z rotace $v = \frac{2\pi r}{T} = \frac{2 \cdot 3,14 \cdot 150 \cdot 10^9}{31\,536\,000}\ \text{m s}^{-1} \doteq \underline{29\,871\ \text{m s}^{-1}}$

- určíme $M_\odot = \frac{v^2 \cdot r}{\kappa} = \frac{29\,871^2 \cdot 150 \cdot 10^9}{6,67 \cdot 10^{-11}}\ \text{kg} \doteq \underline{2 \cdot 10^{30}\ \text{kg}}$

a) $a_g = \frac{\kappa M_\odot}{r^2} = \frac{6,67 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{(150 \cdot 10^9)^2} \doteq \underline{5,94 \cdot 10^{-3}\ \text{m s}^{-2}}$

239/5

$a_g = \frac{1}{2} a_{g0} = \frac{1}{2} g$

$a_g = \frac{\kappa M_Z}{(R_Z + h)^2}$ $\quad a_{g0} = g = \frac{\kappa M_Z}{R_Z^2}$

$h = ?$ (m)

$R_Z = 6378$ km

$\frac{\kappa M_Z}{(R_Z+h)^2} = \frac{1}{2} \frac{\kappa M_Z}{R_Z^2}$

$\sqrt{2} \cdot R_Z = R_Z + h$

$2 R_Z^2 = (R_Z + h)^2 \quad / \sqrt{\ }$

$h = \sqrt{2} R_Z - R_Z = R_Z (\sqrt{2} - 1) = 6378 \cdot 0{,}4142$ km $\doteq$ **2642 km**

239/6

$M_J = 1{,}9 \cdot 10^{27}$ kg

$R_J = 70\,000$ km $= 7 \cdot 10^7$ m

$T_J = 9$ h 50 min $= 35\,400$ s

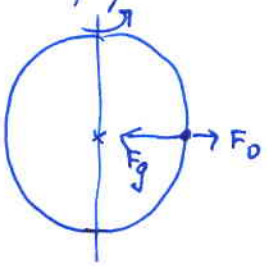
$a_g = ?$ (m s^{-2}) na rovníku planety

$g = ?$ (m s^{-2})

$a_g = \frac{\kappa M_J}{R_J^2} = \frac{6{,}67 \cdot 10^{-11} \cdot 1{,}9 \cdot 10^{27}}{(7 \cdot 10^7)^2}$ m s^{-2} $\doteq 25{,}86$ m s^{-2} $\doteq$

$\doteq$ **25,9 m s^{-2}**

K vektoru $\vec{g}$ si musíme uvědomit, že na těleso na rovníku působí kromě gravitační síly F_g i odstředivá síla F_o. Výsledná síla je dána jejich rozdílem.



$F = F_g - F_o = m \cdot g$ $\quad v = \frac{2\pi R_J}{T_J}$

$\frac{\kappa m \cdot M_J}{R_J^2} - \frac{m v^2}{R_J} = m g$

a_g

$g = a_g - \frac{v^2}{R_J} = a_g - \frac{4\pi^2 \cdot R_J^2}{R_J T_J^2} = a_g - \frac{4\pi^2 \cdot R_J}{T_J^2} =$

$= 25{,}9 - \frac{4 \cdot 3{,}14^2 \cdot 2 \cdot 10^7}{(35\,400)^2} \doteq 25{,}9 - 2{,}2$ m s^{-2} $\doteq$ **23,7 m s^{-2}**

243 / A1

$v_0 = 40\ m s^{-1}$ vrh svislý vzhůru (zpomalený pohyb)

$v = ?\ (m s^{-1})$ $\quad v = v_0 - gt$

$y = ?\ (m)$ $\quad y = h = v_0 t - \frac{1}{2} g t^2$

t (s)	v (m/s)	y (m)
1	30	35
2	20	60
3	10	75
4	0	80

$v_1 = v_0 - g t_1 = 40 - 10 \cdot 1 = 30\ m s^{-1}$

$v_2 = v_0 - g t_2 = 40 - 10 \cdot 2 = 20\ m s^{-1}$

$v_3 = v_0 - g t_3 = 40 - 10 \cdot 3 = 10\ m s^{-1}$

$v_4 = v_0 - g t_4 = 40 - 10 \cdot 4 = 0\ m s^{-1}$

$y_1 = v_0 t_1 - \frac{1}{2} g t_1^2 = 40 \cdot 1 - \frac{1}{2} \cdot 10 \cdot 1^2\ m = 40 - 5\ m = 35\ m$

$y_2 = v_0 t_2 - \frac{1}{2} g t_2^2 = 40 \cdot 2 - \frac{1}{2} \cdot 10 \cdot 2^2\ m = 80 - 20\ m = 60\ m$

$y_3 = v_0 t_3 - \frac{1}{2} g t_3^2 = 40 \cdot 3 - \frac{1}{2} \cdot 10 \cdot 3^2\ m = 120 - 45\ m = 75\ m$

$y_4 = v_0 t_4 - \frac{1}{2} g t_4^2 = 40 \cdot 4 - \frac{1}{2} \cdot 10 \cdot 4^2\ m = 160 - 80\ m = 80\ m$

243 / A2

$v_k = 7{,}0\ km s^{-1}$ pro r

$v_k = \sqrt{\frac{\varkappa M_z}{R_z}} = \sqrt{\frac{\varkappa M_z}{r}}$

a) $r' = 2r$

b) $r' = 4r$

a) $v_k' = \sqrt{\frac{\varkappa M_z}{2r}} = \sqrt{\frac{\varkappa M_z}{r}} \cdot \frac{1}{\sqrt{2}}$

$v_k' = \frac{v_k}{\sqrt{2}} = \frac{7{,}0}{\sqrt{2}}\ km s^{-1} = 4{,}95\ km s^{-1}$

b) $v_k' = \frac{v_k}{\sqrt{4}} = \frac{v_k}{2} = \frac{7}{2}\ km s^{-1} = 3{,}5\ km s^{-1}$

244/3

$h = 20\ m$

$v_0 = ?\ (m s^{-1})$

ve výšce h je $v = 0$

$v = v_0 - g t_h \qquad t_h = \frac{v_0}{g}$

$h = v_0 t_h - \frac{1}{2} g t_h^2$

$h = v_0 \cdot \frac{v_0}{g} - \frac{1}{2} g \cdot \frac{v_0^2}{g^2} = \frac{v_0^2}{g} - \frac{1}{2}\frac{v_0^2}{g} = \frac{1}{2}\frac{v_0^2}{g}$

$2gh = v_0^2 \Rightarrow v_0 = \sqrt{2gh} = \sqrt{2.10.20} = 20\ m s^{-1}$

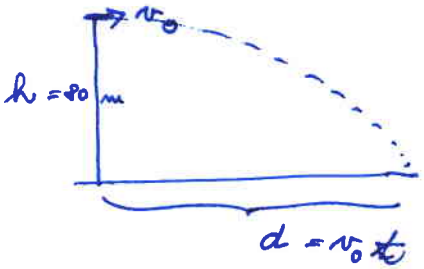
244/5

$h = 80\ m$

$v_0 = 30\ m s^{-1}$

$t = ?\ (s)$

$d = ?\ (m)$



$y = h - \frac{1}{2} g t^2$

$x = v_0 t$

Dobu pádu určíme z podmínky že

$y = 0 \Rightarrow h = \frac{1}{2} g t^2 \qquad t = \sqrt{\frac{2h}{g}}$

$t = \sqrt{\frac{2.80}{10}}\ s = \sqrt{16}\ s = 4\ s$

$d = v_0 . t = 30 . 4\ m = 120\ m$

244/6

$v_k' = \frac{1}{2} v_k$

$h = ?$

$v_k = \sqrt{\frac{\varkappa M_z}{R_z}} \qquad v_k' = \sqrt{\frac{\varkappa M_z}{(R_z + h)}}$

$\sqrt{\frac{\varkappa M_z}{(R_z+h)}} = \frac{1}{2}\sqrt{\frac{\varkappa M_z}{R_z}} \quad / ^2$

$\frac{\varkappa M_z}{(R_z+h)} = \frac{1}{4}\frac{\varkappa M_z}{R_z}$

$4 R_z = (R_z + h)$

$h = 4R_z - R_z$

$h = 3 R_z$